TEMPERATURE DISTRIBUTION AND HEAT-TRANSFER COEFFICIENT DURING CONDENSATION OF STEAM ON A RECTANGULAR FIN

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Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 6, pp. 853–859, 1967

UDC 536.21

The distribution of temperature with height and length of a rectangular fin in the condensation of vapor has been found theoretically. The effectiveness of fins under different conditions is demonstrated.

In the condensation of pure steam, containing no impurities, the process of heat transmission through the wall is limited, as a rule, by the cooling heat transfer agent, water. This occurs in fact during condensation of water vapor and vapors of other substances with comparatively large latent heat of vaporization, high thermal conductivity, and low viscosity in the liquid state. However, in a number of cases, for example during the condensation of vapors of low-boiling organic substances, whose physical properties do not satisfy the above criteria, the intensity of heat transfer on the vapor side is appreciably decreased. Thus, the question of the desirability of developing surfaces on the condensation side, by the application of finning, is to be considered.

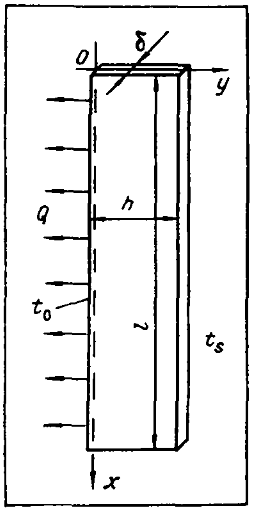

Fig. 1. Tube element with longitudinal finning.

This paper solves the heat-conduction equation for a rectangular fin considered as an element of a longitudinally finned tube, under the conditions of film condensation of the vapor.

We shall consider a vertically positioned plane rectangular fin, located in a vapor medium at temperature t_s. The heat liberated by the vapor upon condensation at the fin surface is given out through the base of the fin, which has a temperature t_0 which is below t_s (Fig. 1). We are required to find the distribution of the temperature with fin height and length.

Solution of this problem is simplified if we neglect the variation of temperature with thickness of the fin. This assumption will be close to reality when the fin thermal conductance is large and the fin thickness is small. As a result we obtain a one-dimensional differential equation for the propagation of heat over the fin height:

$$\frac{d^2T}{dy^2} = \frac{\alpha u}{\lambda_w f} T. \tag{1}$$

The boundary conditions in the most general case take the form

$$\text{for } y = 0 \quad T = T_0 = t_s - t_0,$$
$$\text{for } y = h \quad -\lambda_w \frac{dT}{dy} = \alpha T. \tag{2}$$

The value of the heat-transfer coefficient for fin vapor condensation on a vertical plate cooled at its root, which value must be substituted into Eq. (1) to obtain a solution, depends entirely on the thickness of the condensate film formed on the fin surface.

The problem of the runoff film thickness and of the heat-transfer coefficient on a vertical wall with constant temperature was examined by Nusselt. Under conditions of steady laminar flow, neglecting inertia and surface tension forces, he obtained the widely known relation for the heat-transfer coefficient,

$$\alpha = \sqrt[4]{\frac{\lambda_l^3 \gamma_l^2 r}{4\mu_l (t_s - t_w) x}}. \tag{3}$$

An essential difference between conditions for condensation on a fin and the conditions of the Nusselt problem is that the film thickness on the fin changes not only along the length of the fin from top to bottom, as on a plate, but also over the height of the fin, from root to apex, because of temperature variation.

However, if we neglect the wall temperature variation along every imaginary vertical line of motion of the film condensate, and neglect the mutual interaction of neighboring liquid layers over the height of the fin in a film moving with a different velocity, we can use the Nusselt equation (3) and substitute it into the basic equation (1).

Taking account of the above considerations, and using the notation

$$\theta = \frac{T}{T_0}; \quad \xi = \frac{y}{h};$$

$$m = \frac{0.71\lambda_l^{3/4}\gamma_l^{1/2} r^{1/4} u h^2}{\mu_l^{1/4} x^{1/4} \lambda_w f T_0^{1/4}}, \tag{4}$$

we reduce the equation of heat conduction over the fin [Eq. (1)] to the dimensionless form

$$\frac{d^2\theta}{d\xi^2} = m\theta^{3/4} \tag{5}$$

with the boundary conditions

$$\text{for } \xi = 0 \quad \theta = 1,$$

$$\text{for } \xi = 1 \quad \frac{d\theta}{d\xi} = -n\theta^{3/4},$$

$$n = \frac{0.71\lambda_l^{3/4}\gamma_l^{1/2} r^{1/4} h}{\mu_l^{1/4} x^{1/4} \lambda_w T_0^{1/4}}. \tag{6}$$

Thus, we have obtained a nonlinear second-order differential equation. The coefficient m on the right side of Eq. (5) contains two thermophysical properties of the fin material and working substance in the liquid phase, the geometrical dimensions of the fin, and a variable height coordinate ($0 \le x \le l$).

Equation (5) can be reduced to first order by substituting $d\theta/d\xi = p$, and then the first integral is

$$\frac{d\theta}{d\xi} = -\sqrt{\frac{8}{7} m\theta^{7/4} + C_1}. \tag{7}$$

On the right of the new equation (7) we retain only the "minus" sign, since the sense of the problem requires us to find a decreasing function.

The constant C_1 is determined from the boundary conditions (6) at the fin apex

$$C_1 = n^2\theta_h^{6/4} - \frac{8}{7} m\theta_h^{7/4}, \tag{8}$$

where $\theta_h = \theta|_{\xi=1}$.

Separating the variables in Eq. (7)

$$\frac{d\theta}{\theta^{7/8}\left(1 + \frac{7C_1}{8m}\theta^{-7/4}\right)^{1/2}} = -1.07\sqrt{m}\, d\xi \tag{9}$$

and expanding in a series the expression in brackets on the left of Eq. (9) with respect to $(7C_1/8m)\theta^{-7/4}$

$$\left(1 + \frac{7C_1}{8m}\theta^{-7/4}\right)^{-\frac{1}{2}} = 1 - \frac{1}{2}\frac{7C_1}{8m}\theta^{-7/4} +$$

$$+ \frac{1\cdot 3}{2\cdot 4}\left(\frac{7C_1}{8m}\right)^2 \theta^{\frac{-7\cdot 2}{4}} - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\left(\frac{7C_1}{8m}\right)^3 \theta^{\frac{7\cdot 3}{4}} + \dots, \tag{10}$$

we can integrate Eq. (9) term by term. We note first that series (10) converges in the region

$$\left|\frac{7C_1}{8m}\theta^{-7/4}\right| = \left|\frac{n^2\theta_h^{6/4} - \frac{8}{7} m\theta_h^{7/4}}{\frac{8}{7} m\theta^{7/4}}\right| \leqslant$$

$$\leqslant \left|\frac{n^2 - \frac{8}{7} m}{\frac{8}{7} m}\right| = |\varphi m - 1| < 1, \tag{11}$$

where $\varphi = (7/8)(n^2/m^2) = (7/8)(\delta/2h)^2$ is a coefficient depending on the fin dimensions.

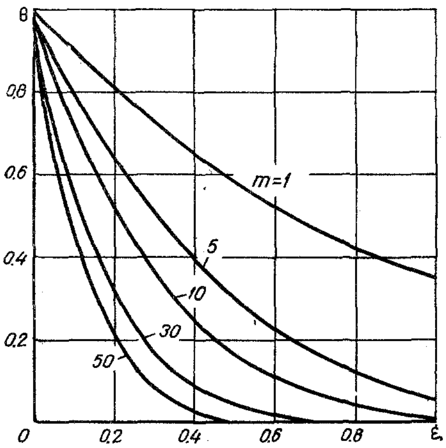

Fig. 2. Temperature distribution over the height of a rectangular fin, for various values of m.

Convergence of the series is restricted to the range $m < 2/\varphi$. Since $m \sim x^{-1/4}$, the parameter m begins to increase noticeably for $x \to 0$, i.e., at the top edge of the fin, where the convergence of the series (10) also may break down. Calculations carried out for various variants of real surfaces and working substances indicate that condition (11) is satisfied right up to $x = 10^{-5}$ m. In practice this means that the series (10) converges over the whole range of values $0 \le x \le l$.

After substituting series (10) into Eq. (9) and integrating, we have

$$8\theta^{1/8} + 0.30\frac{7C_1}{8m}\theta^{-\frac{13}{8}} - 0.111\left(\frac{7C_1}{8m}\right)^2\theta^{-\frac{27}{8}} +$$

$$+ \dots = -1.07\sqrt{m}\xi + C_2. \tag{12}$$

We shall restrict ourselves to the first term of the expansion on the left of Eq. (12), since the subsequent terms are negligibly small. To this level of accuracy, we find that $C_2 = 8$ (for $\xi = 0$ $\theta = 1$).

Thus, the approximate equation obtained for the temperature distribution across the fin is

$$\theta = (1 - 0.133\sqrt{m}\xi)^8, \quad 0 \leqslant m \leqslant 56. \tag{13}$$

It follows from the form of Eq. (13) that it satisfies the physical sense of the problem only in a limited range of values of the parameter m. (For m > 56, according to Eq. (3), the fin wall temperature increases with distance from the root.)

The graph of the function θ, shown in Fig. 2, gives the temperature distribution over fin height for various values of m. It can be seen that Eq. (13) covers rather well the range of interest to us. For low values of m ($m \le 1$), which corresponds to poor heat transfer, the temperature drop over the height of the fin is smooth and the temperature curves approximate to horizontal lines in the limit, corresponding to m = 0. Even for m > 30, the main temperature drop occurs in the section (0.2–0.3), while the greater part of the fin practically plays no part in the condensation.

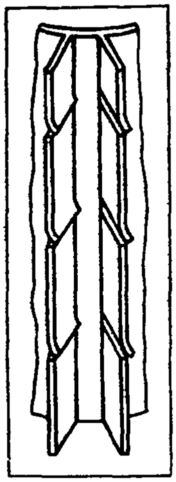

Fig. 3. Intermittent longitudinal tube finning.

Thus, the most effective region, from economic and technical considerations, is the region of values of the parameter m = 1–10, for which the entire fin surface operates with the greatest heat transfer.

Using the expression obtained [Eq. (13)] for the temperature distribution over a fin during condensation of the vapor, we determine the mean dimensionless temperature gradient over the fin height,

$$\tilde{\theta} = \int_0^1 \theta\, d\xi = \frac{(1-0.133\sqrt{m})^9}{1.2\sqrt{m}}. \tag{14}$$

From Eq. (14), taking into account Eq. (4), i.e., considering that $m \sim h^2$, we can see that $\tilde{\theta}$ depends strongly on the fin height.

The mean-integral temperature drop over the whole fin area is

$$\bar{T} = \frac{T_0}{l}\int_0^l \tilde{\theta}\, dx = \frac{0.9T_0}{a}\left[\frac{l^{9/8}}{9} - \frac{al}{8} + \frac{a^2 l^{7/8}}{7} - \ldots + a^8 l^{1/8} - a^9 \ln\frac{l^{1/8}+a}{a}\right], \tag{15}$$

where

$$a = 0.133\sqrt{m}\, x^{1/8} = \frac{0.10\lambda_l^{3/8}\gamma_l^{1/4} r^{1/8} u^{1/2} h}{\mu_l^{1/8}\lambda_w^{1/2} f^{1/2} T_0^{1/8}}.$$

The amount of heat passing through the fin base is

$$Q = -\frac{\lambda_w \delta T_0}{h}\int_0^l \left.\frac{d\theta}{d\xi}\right|_{\xi=0} dx = \frac{1.43\lambda_l^{3/8}\lambda_w^{1/2}\gamma_l^{1/4} r^{1/8} T_0^{7/8} l^{7/8}\delta^{1/2}}{\mu_l^{1/8}}. \tag{16}$$

Finally, the mean heat-transfer coefficient over the fin surface is

$$\tilde{\alpha} = \frac{Q}{lh\bar{T}}. \tag{17}$$

The quantities Q and T appearing in Eq. (17) are determined from Eqs. (16) and (15), respectively.

The work performed, making use of the formulas obtained, allows us to make a specific estimate of the effectiveness of various kinds of longitudinal finning, as a function of their operating conditions.

As an example, we consider the frequently used finned aluminum tubes with Ø 24/22, of height 1 m, positioned vertically in condensers (Table 1).

A calculation of the fin efficiency was carried out for the case of the condensation of two substances, water and carbon tetrachloride, which differ markedly in their thermophysical properties. We calculated the heat transfer from a smooth tube by means of the Nusselt formula. Calculation of the heat transfer in the case of a fin was carried out according to Eq. (13).

Table 1

Geometrical Characteristics of Finned Surfaces

Type of surface	Fin length, m	Fin height, m	Fin thickness, mm	Number of fins around the tube circumference
1. Longitudinally continuous fins	1.0	7	0.6	12
2. Longitudinally intermittent fins (Fig. 3)	0.05	7	0.6	12

The results of the calculations, shown in Table 2, allow us to conclude that in spite of the high thermal conductance of the chosen (fin) materials, the considered fin shapes are not suitable for the condensation of water vapor, but they do have an appreciable effect on the condensation of the vapors of organic substances. This is particularly noticeable for the second type of tube finning with distributed longitudinal fins, where the 68% increase in weight of a finned tube in comparison

Table 2

Technical and Economic Data Relevant to the Fins*

Working substance	r, kJ/kg	Q_s, kW	1: Q_f, kW	1: $\frac{Q_f}{Q_s}$100, %	1: $\frac{G_f}{G_s}$100, %	2: Q_f, kW	2: $\frac{Q_f}{Q_s}$100, %	2: $\frac{G_f}{G_s}$100, %
H_2O	2250	47	21	44	68	30	59	68
CCl_4	192	4.7	6.1	135	68	9.2	202	68

* 1 and 2 denote the surface types (see Table 1).

with a smooth tube leads to an increase of 200% in heat transfer (without considering heat removed by the intervening parts of the tube).

Thus, we have obtained an approximate solution to the problem of the temperature distribution with height and length of a vertically positioned rectangular fin, during the condensation of an impurity-free vapor under steady conditions and laminar condensate flow. Using Eq. (13), we can answer the question of the desirability of employing cooling surfaces with longitudinal fins in condensers, of choosing the most efficient fin dimensions, as indicated by the thermophysical characteristics of the fin material and the condensing vapor, and of calculating the total heat-transfer coefficient.

NOTATION

x, y are the fin length and height coordinates, respectively; t_s is the vapor saturation temperature; t_w is the fin wall temperature; t_0 is the fin root wall temperature; h, l, δ, are the fin height, length, and thickness, respectively; $T = t_s - t_w$ is the local temperature drop between the vapor and the fin wall; u is the perimeter of the fin section taking part in heat exchange with the vapor; f is the fin cross sectional area; λ_w is the thermal conductivity of the fin; λ_l is the thermal conductivity of the liquid; μ_l is the dynamic viscosity of the liquid; γ_l is the weight of liquid per unit volume; r is the heat of vaporization of the liquid; θ is the dimensionless temperature gradient at the fin; $\widetilde{\theta}$ is the mean-integral dimensionless temperature gradient over the fin height; $\overline{T}$ is the mean-integral temperature drop over the total fin area; Q is the amount of heat passing through the fin base; G_s and G_f are the weight of the smooth tube and the finning, respectively; Q_s and Q_f are the heat given out by the smooth tube and the finned tube, respectively.

29 March 1967

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